SHEET NO (1)

Q1)

Consider two random variables X and Y with a joint probability mass function $p(x,y)$ and marginal probability mass functions $p(x)$ and $p(y)$. Let I(X;Y) be the mutual information between X and Y. Prove that:

a. $I(X; Y) = H(Y) - H(Y|X) = H(X) - H(X|Y) = I(Y; X)$
b. $I(X; Y) = H(Y) + H(X) - H(X,Y)$
c. $I(X; X) = H(X)$
d. $I(X;Y) \geq 0$ with equality if and only if X and Y are independent

Q2)

Consider the two random variables X and Y with a joint probaiblity P(x, y) given as shown below.

$P(x,y)$		x			
		1	2	3	4
y	1	1/8	1/16	1/32	1/32
	2	1/16	1/8	1/32	1/32
	3	1/16	1/8	1/16	1/16
	4	1/4	0	0	0

a. Find the value of H(X, Y).
b. Find the value of H(X) and H(Y).
c. For each value of Y find the conditional entropy $H(X|Y = y)$.
d. Find the value of $H(X|Y)$.
e. Find the value of $I(X; Y)$.
f. Find the value of Kullback-Leibler Divergence between X and Y.

Q3)

If X is a random variable with Alaphbet $A_x = \{x_1, x_2, \ldots . x_n\}$ then prove that:

a. $H(X) \leq \log(n)$ with equality if and only if X is uniform random variable.
b. $H_b(X) = log_b(a)\, H_a(X)$.
c. If $p(x)$ and $q(x)$ are two probablitiy distribution definded over the same alaphbet A_x then prove that the Kullback-Leibler Divergence $D\big(p(x)||q(x)\big) \geq 0$ with equality if and only if $p(x) = q(x)$.

Q4)

True or False? If the inequality is true, prove it, otherwise, give a counterexample:

(a) $H(X,Y|Z) \geq H(X|Z)$

(b) $H(X|Z) \leq H(Z)$

(c) $H(X,Y,Z) - H(X,Y) \geq H(X,Z) - H(X)$

(d) $H(X|Z) \leq H(X) - H(Z)$

Q5)

Let $p(x, y)$ be given by

X \ Y	0	1
0	$\frac{1}{3}$	$\frac{1}{3}$
1	0	$\frac{1}{3}$

Find:

(a) $H(X)$, $H(Y)$.

(b) $H(X \mid Y)$, $H(Y \mid X)$.

(c) $H(X, Y)$.

(d) $H(Y) - H(Y \mid X)$.

(e) $I(X; Y)$.

(f) Draw a Venn diagram for the quantities in parts (a) through (e).